Arithmetic and Geometric Progression

YORK COLLEGE & UNIVERSITY CENTRE

Sequences & Series

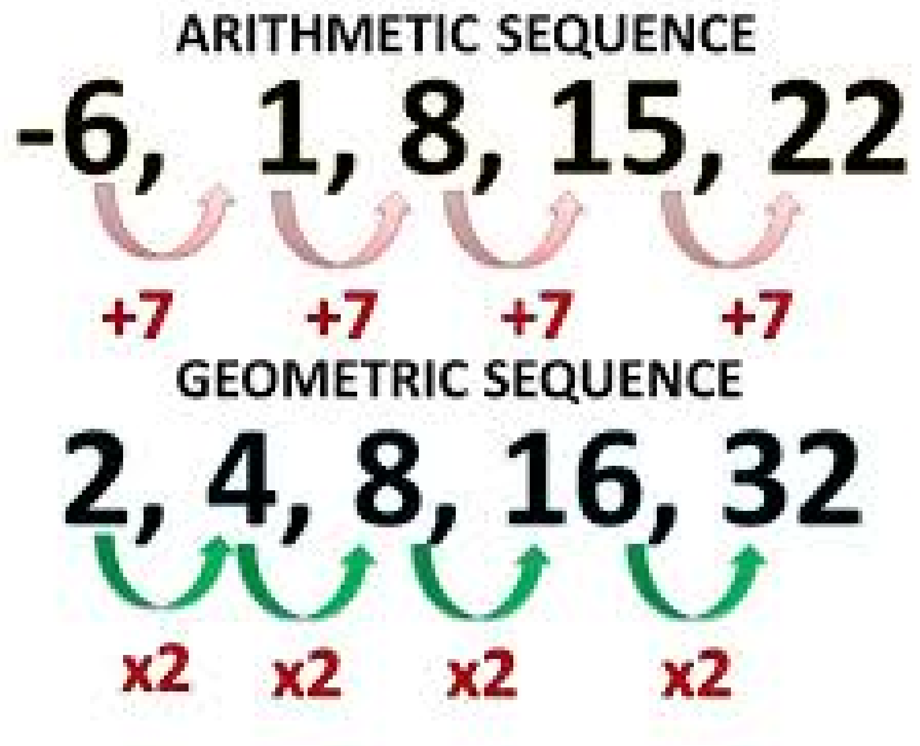
- A sequence is a list of terms separated by commas
  - 2, 4, 6, 8, 10, 12
  - It has the notation n terms for the location in the list n=1, n=2, n=3 ect.
  - It has the notation a terms for the value in that list $a_1=2$, $a_2=4$, $a_3=6$

- A series is a sum, you are adding up all the terms in the list
  - 2+4+6+8+10+12

Arithmetic vs Geometric

- Arithmetic sequences add the same thing each time to get to the next term.

- Geometric sequences multiply by the same thing each time to get to the next term.



Are These Sequences Arithmetic or Geometric

- 1, 5, 9, 13, 17…
- 10, 40, 160, 640…
- 1024, 962, 900, 838…
- x, 8x, 64x, 512x…
- 2x, 2x-3, 2x-6, 2x-9…
- $a_n = 2+4(n-1)$
- $a_n = 2(4)^{n-1}$

Finding a Value in an Arithmetic Sequence

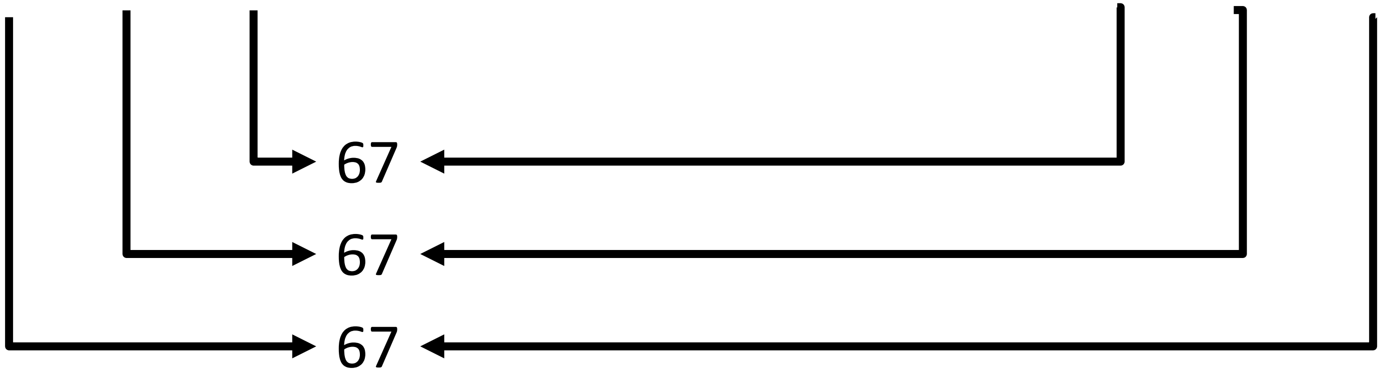
- We want to find the 6th value in this sequence
- 8, 12, 16, 20, 24, ?
- Common difference between values (d) = 12-8 = 4
- By the 6th value how many times will we have added 4? (5 times)
- We can take our first value and then add on all the times we would have added 4
- 8 + 4 + 4 + 4 + 4 + 4 = 8+4(5) = 28
- We can reverse engineer our equation for the value in an arithmetic sequence as $a_n = a_1 + d(n-1)$

Finding a Value in a Geometric Sequence

- We want to find the 9th value in this sequence
- 12, 36, 108, 324, …, 26244, ?
- Common ratio between values (r) = 36/12 = 3
- By the 9th value how many times will we have multiplied by 3? (8 times)
- We can take our first value and then multiply by the common ratio for the number of times.
- 12 * 3 * 3 * 3 * 3 * 3 * 3 * 3 * 3 = $12(3)^8$
- We can reverse engineer our equation for the value in an geometric sequence as $a_n = a_1(r)^{n-1}$

Finding the Sum of an Arithmetic Series

- 6 + 11 + 16 + 21 + 26 + 31 + 36 + 41 + 46 + 51 + 56 + 61



- When finding the value of a series we could add up each individual value but for long series that could take ages
- We know that pairs of numbers around the median add up to a number but how pairs are there?
- n/2

Finding the Sum of an Arithmetic Series

- 6 + 11 + 16 + 21 + 26 + 31 + 36 + 41 + 46 + 51 + 56 + 61

67

- How do we find the difference between pairs?
- We can work out the second number by adding n-1(d) to the first number so in this example 6 + 12-1(5) = 61
- To work out the sum of that pair we can do 2x the initial number plus the gap between the two numbers for example (2*6)+12-1(5) = (2*6)+55 = 67
- This times by the n/2 gives us our sum value $\frac{n}{2}\left(2a_1 + (n-1)d\right)$
- So in this example: $\frac{12}{2}\left(2 \cdot 6 + (12-1)5\right) \quad = 402$

Finding the Sum of a Geometric Series

- To find the sum of a geometric sequence we use this formula:

$$s_n = \frac{a_1\left(1 - r^n\right)}{1 - r}$$

- So for the sequence 12, 24, 48, 96, 192
- We do $s_n = \frac{12\left(1 - 2^5\right)}{1 - 2}$ as n = 5, r = 2, a_1 = 12
- This gives us 372

Equations

	Arithmetic	Geometric
Sequence	$a_n = a_1 + d(n-1)$	$a_n = a_1(r)^{n-1}$
Series	$s_n = \frac{n}{2}\left(2a_1 + (n-1)d\right)$	$s_n = \frac{a_1\left(1-r^n\right)}{1-r}$